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On sectoral market efficiency

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ABSTRACT

A multi-fractional Brownian approach is used to measure the level of sectoral market efficiency through the Hurst exponent, using S&P 500 and sectoral indices data between 2002 and 2022. Our results show that each sector has a particular level of market efficiency, and it cannot be statistically represented by the aggregate market efficiency. However, there are long and short-term relationships between the efficiency of each sector and the level of market efficiency, which tend to vary from one sector to another. Besides, during periods of crisis, market efficiency by sector decreases sharply, and the cross-correlation of efficiency between sectors tends to increase. On the other hand, during the bull periods, the market efficiency could be considered a good hypothesis for the different sectors.

1. Introduction

Since the seminal work of Fama (1970), most economists claim that market prices "fully reflect" the available information, premise known as the Efficient Market Hypothesis (EMH). In fact, the most purist economic logic suggests that a piece of information about the value of a share will not affect its price, since if the share price does not reflect that information, investors will trade the asset until the price is in equilibrium, i.e. when the information is no longer useful for trading. According to this point of view stock prices can be considered as unpredictable.

In this context, the EMH is usually linked to the random walk theory. In fact, from a mathematical point of view, the EMH implies that stock prices follow a Brownian motion dynamics, one of the cornerstones of the fundamental theorem of asset pricing. In simple terms, if arbitrage is impossible, then the price of a stock is the discounted value of its future price. Assuming the stochastic discount factor is constant, and the time interval is short enough so no dividend is paid, the price dynamics could be modeled as:

$$P_t = M E_t[P_{t+1}]$$

where E_t is the expected value given information at time t, and M is the stochastic discount factor, which implies that the log of stock prices follows a random walk (with a drift).

However, the EMH has long been challenged from different perspectives, since it is based on several unrealistic assumptions such as: normality of returns, homoskedasticity, serial independence, and the absence of long memory. One early criticism, developed by Mandelbrot (1963), Mandelbrot and Taylor (1967), was the proposition that financial market returns in fact possess long memory properties.

Indeed, one of the main reasons why an efficient market does not exist in the real world is because of the tendency of markets to move back to the mean value of the index over a significantly long period of time. Since markets tend to move toward the mean, participants with quality information can outperform the efficient market hypothesis. This happens because of the lack of complete

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arbitrage of the information available in the market. Therefore, long-term memory is intuitively a characteristic of less developed financial markets, as opposed to efficient markets.

A popular way to empirically test whether an asset's returns follow a random walk, and to analyze its long-term memory properties, is to calculate the Hurst exponent, using the fractional Brownian motion (fBM) model (Hurst, 1951; Mandelbrot and Van Ness, 1968). Actually, the Hurst exponent provides a measure of the long-term memory and fractality of a time series. The values of the Hurst exponent range from 0 to 1, and can belong to three categories: (i) $H = 0.5$ indicating a random walk, and therefore the market is efficient, (ii) $0 < H < 0.5$ indicating an anti-persistent series, meaning that a rising value is more likely to be followed by a falling value, and vice versa. Here, the series tends to become rougher, as H approaches 0. (iii) $0.5 < H < 1$ indicates a persistent series, meaning that the direction (upward or downward) of the next value is more likely to be the same as that of the current value. Here, the series tends to smooth out more, as H increases.

In recent times, however, researchers have found a time-varying behavior of the Hurst coefficient of H, in particular in financial time series, as indicated by the work of Vogl (2023) and Assaf et al. (2022), among others. One way to account for the dynamics of H is through the multifractional Brownian motion (mBM) introduced by Peltier and Véhel (1995), which relaxes the assumption of constant H values, and at the same time acts locally as an fBM, which ensures the consistency of the analysis with the previous paragraph.

In this paper, a multi-fractional Brownian approach is used to measure the level of sectoral market efficiency through the Hurst exponent, using S&P 500 and sectorial indices data between 2002 and 2022. Our main objective is to analyze the dynamics of market efficiency for the different sectors present in the S&P 500, and how this efficiency could be explained from the aggregate market efficiency. In addition, we develop different stylized facts for sectoral efficiency, which leads us to believe that this indicator should be continuously monitored, given the notorious regularities we detect.

The paper is organized as follows: Section 2 presents a brief literature review of the use of multi-fractional Brownian approach used to measure the EMH in the last years. Section 3 shows the model to be calibrated with the S&P 500 data. Section 4 presents the data used in the analysis. Section 5 develops the analysis of the results, which consider a graphical, statistical, and econometric analysis. Finally, in Section 6 the conclusions are discussed.

2. Literature review

Nowadays, the study of time variations of Hurst exponents constitutes an important research agenda in this area of finance. Indeed, there is a growing literature using the fractional Brownian motion (fBM) as a model of the price dynamics, in particular with the aim of testing the dynamics of the EMH. For example, Frezza et al. (2021) analyze the impact of the COVID-19 pandemic on the efficiency of fifteen financial markets from Europe, US and Asia. They found that the inefficiency that characterizes US and European markets originates moderately high levels of volatility. In the same line, Gaio et al. (2022) analyze the impact of the Russia-Ukraine conflict on the stock market efficiency of six developed countries. Their analysis rejects the market efficiencyhypothesis and indicates the predictability of asset prices in times of instability and global financial crisis. Kakinaka and Umeno (2022) study market efficiency of the major cryptocurrencies during the COVID-19 pandemic, accounting for different investment horizons. They found that after the outbreak, the markets exhibited stronger multifractality in the short-term but weaker multifractality in the long-term. Similarly, Arouxet et al. (2022), through a wavelet-based Hurst estimator, found an strong temporary impact on cryptocurrency volatilities. These results confirm that the pandemic has greatly changed the cryptocurrency markets. Le Tran and Leirvik (2019) construct a new measure to quantify the level of market efficiency over time. They find that markets are often efficient, but can also be significantly inefficient over longer periods. Navratil et al. (2021) study the COVID-19 pandemic, and the efficiency of the United States equity market. They showed that utility maximizing agents generated statistically significant profits during that time, utilizing only historical price and virus related data. On the other hand, Buonocore et al. (2020) discuss the importance of multifractality (i.e., processes which are not fully characterized by a single Hurst exponent) in financial markets. To get a comprehensive understanding of the current state-of-the-art,

Table 1 summarizes the four most common approaches (mBm, multi-fractal detrended moving average, generalized Hurst exponents, and wavelet-based) to determine the time-varying Hurst exponents and its implications on market efficiency.

3. The model

The fractional Brownian motion (fBM) is a stochastic process with correlated increments, according to the values of the Hurst exponent $H \in (0,1)$, which is constant along its paths. As we mentioned above, when $H = 1/2$, the fBm is reduced to the Brownian motion. A constant H along the paths of the fBm represents an important drawback in modeling financial time series whose point-wise regularity changes. In this context, the multifractional Brownian motion (mBm) constitutes a major development, since it replaces the constant Hurst index by a Hölder function $H : [0,\infty) \to (0,1]$.

Several methods have been proposed to estimate $H(t)$ in the literature, most of them based on adaptations of asymptotic estimators available for the fBm. They generally involve second order variations statistics. In this work, we will use in our investigation the absolute moment based estimator (AMBE) technique presented by Bianchi et al. (2013), Bianchi (2005). They propose an unbiased, low-variance moving window estimator $\hat{H}$, that depends upon, δ, the size of the moving window, q the differencing lag, and n the sample's size. $\hat{H}$ is unbiased and normally distributed, and its variance can be determined analytically. Similar representations can be found in Frezza et al. (2021), Pianese et al. (2018), Bianchi and Pianese (2018).

Table 1
Summary review for different approaches to determine time-varying Hurst functions. Selected publications and findings are included to describe these research directions.

Method/Approach	Selected references	Main findings
Multifractional Brownian motion	Bianchi et al. (2013)	Hurst exponent fluctuates around the half with large deviation in turbulent periods
	Bianchi and Pianese (2018)	The market alternate periods of efficiency and inefficiency. Negative inefficiency (H<1/2) tends to occur simultaneously on different markets.
	Frezza et al. (2021)	COVID-19 crisis reflects large negative inefficiency.
Multifractal dentrented moving average	Horta et al. (2014)	Crisis periods leads to a significant increase in the local Hurst exponent correlations over different geographical markets
	Jin (2016)	Inefficiency is detected during the 2008 financial crisis period plus a significant rise in the local Hurst exponent correlations on Asian markets.
	Al-Yahyaee et al. (2018)	Efficiency is rejected for Gold, the global stock market index, the US dollar index, and Bitcoin. The latter is significantly the most inefficient.
	Mensi et al. (2021)	Green bonds market presents time-varying (in)efficiency which depends on upward or downward trends.
Generalized Hurst exponent	Hiremath and Narayan (2016)	Long memory persistence in the Indian market.
	Jiang et al. (2018)	Long-term memory in Bitcoin market.
	Tiwari et al. (2021)	Time-varying efficiency in crude-oil and related products. The persistence increase after the 2008 financial crisis.
Wavelet	Arouxet et al. (2022)	Covid-19 slightly affected the long memory of 7 cryptocurrency returns. However, the impact is severe on volatility long memory.
	Assaf et al. (2022)	Alternancy between high and low persistence in the Hurst dynamics for six cryptocurrencies. High indicator of inefficiency for LTC, ETH, XRP, XMR. and DASH.
	Jena et al. (2022)	Hurst index varies on time-series of six cryptocurrencies. DASH and NEM rank as the most inefficient markets, while ETH and XRP were sorted as the most efficient.

The estimator $\hat{H}$ is defined as:

$$\hat{H} = -\frac{\log\left(\sqrt{\pi}S^k / \left(2^{k/2}\Gamma\left(\frac{k+1}{2}\right)K^k\right)\right)}{k\log\left(\frac{n+1}{q}\right)} \tag{1}$$

with

$$S^k = \frac{1}{\delta - q + 1}\sum_{j=1-\delta}^{i-q}\left|X_{j+q,n} - X_{j,n}\right|^k$$

and $i = \delta + 1, \dots, n$.

The values for k, δ and q are selected according the arguments from Bianchi and Pianese (2018): $\delta = 21$, $q = 1$, and $k = 2$. The procedure to determine unknown K, which control the bias of the estimator, is described in detail by Bianchi et al. (2013) and involves a lineal adjustment in the plane $(x, y) = \left(\log\left(\frac{u}{1-n}\right), \mathbb{E}\left|X_{j+u} - X_j\right|^2\right)$, for increasing values of u. The estimated regression $y_u = \hat{m}x_u + \hat{b}$ reveals K through the intercept: $\exp\left(\hat{b}/2\right) = K$.

As described by Frezza et al. (2021) the variance of the estimator is given in close-form for $H(t) = 1/2$, and then one can define a confidence interval, where the estimator is not different than the martingale condition. Thus, $\hat{H}$ is non-statistical different than 1/2, at the 95% level of confidence, if it's belong to the interval $\left[0.5 - 1.96\sigma_{1/2}, 0.5 + 1.96\sigma_{1/2}\right]$ with $\sigma_{1/2} = \sqrt{\frac{\sqrt{\pi}\Gamma\left(\frac{2k+1}{2}\right) - \Gamma^2\left(\frac{k+1}{2}\right)}{\delta k^2(n-1)\Gamma^2\left(\frac{k+1}{2}\right)}}$.

4. Data

The adjusted daily closing values of the S&P 500 market index, and the different economic sectors represented in the S&P 500 are compiled for the period October 9, 2002 through July 13, 2022. In particular, the 11 sectors considered are: Energy (ENY), Materials (MAT), Industrials (IND), Consumer Discretionary (DIS), Consumer Staples (STA), Health Care (HC), Financials (FIN), Information Technology (TEC), Telecommunication Services (COM), Utilities (UTI), and Real State (RE). Besides, the bullish and

Table 2
S&P 500 Bear and Bull markets periods (10/09/2002–07/13/2022).

Period label	Initial date	Final date	Return (%)	Classification
a	Oct 9, 2002	Oct 9, 2007	101.5	Bull market
b	Oct 9, 2007	Mar 9, 2009	−56.8	Bear market
c	Mar 9, 2009	Feb 19, 2020	400.5	Bull market
d	Feb 19, 2020	Mar 23, 2020	−33.9	Bear market
e	Mar 23, 2020	Jan 3, 2022	114.4	Bull market
f	Jan 3, 2022	Jul 13, 2022	−21.8	Bear market

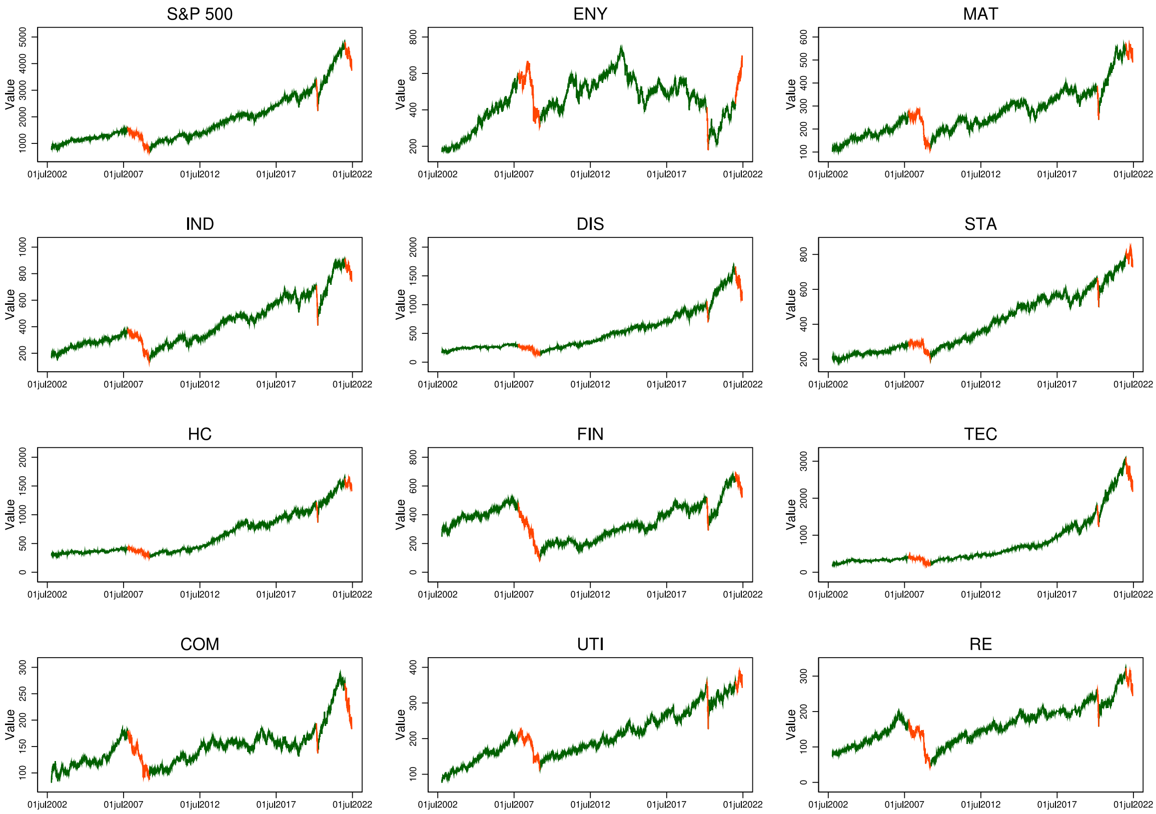


Fig. 1. Values for S&P 500 and sectorial indexes in the studied period. Green (red) lines indicates Bull (Bear) market regime.

bearish periods of the indexes are identified. A bull (bear) market is defined[1] as a 20% increase (decrease) from the previous low (high), which ends when the market (in our case the SP500 index) reaches a high (low) and subsequently falls (rises) by 20%.

From Table 2 we can see that distress periods b and d corresponds to major economic crashes, namely the 2007–2008 global financial crisis, and the Covid-19 pandemic, respectively; while the decline period f is in line to instabilities related to inflation surge, Russian-Ukrainian war, and the global supply chain crisis. Fig. 1 shows the value for each analyzed index according to the market Bear or Bull state.

5. Results

5.1. Graphical analysis

The Hurst exponent is calculated using Eq. (1) for all these indexes. Fig. 2 shows the time series graphs of Hurst exponents for each of the sectors studied. Firstly, we can observe significant deviations from the efficient market hypothesis in all sectors.

[1] Even though there is no strict definition for bull and bear periods, there is a common consensus about bull (bear) markets reflects periods of rising (falling) prices (Chauvet and Potter, 2000). In particular we follow the ex-post 20% rule of thumb on capital gain (loss) over a holding period (Sperandeo, 1994; Pagan and Sossounov, 2003; Lunde and Timmermann, 2004). This approach is widely used for financial press to define bull and bear time-spams (Bloomberg, 2023; Reuters, 2023).

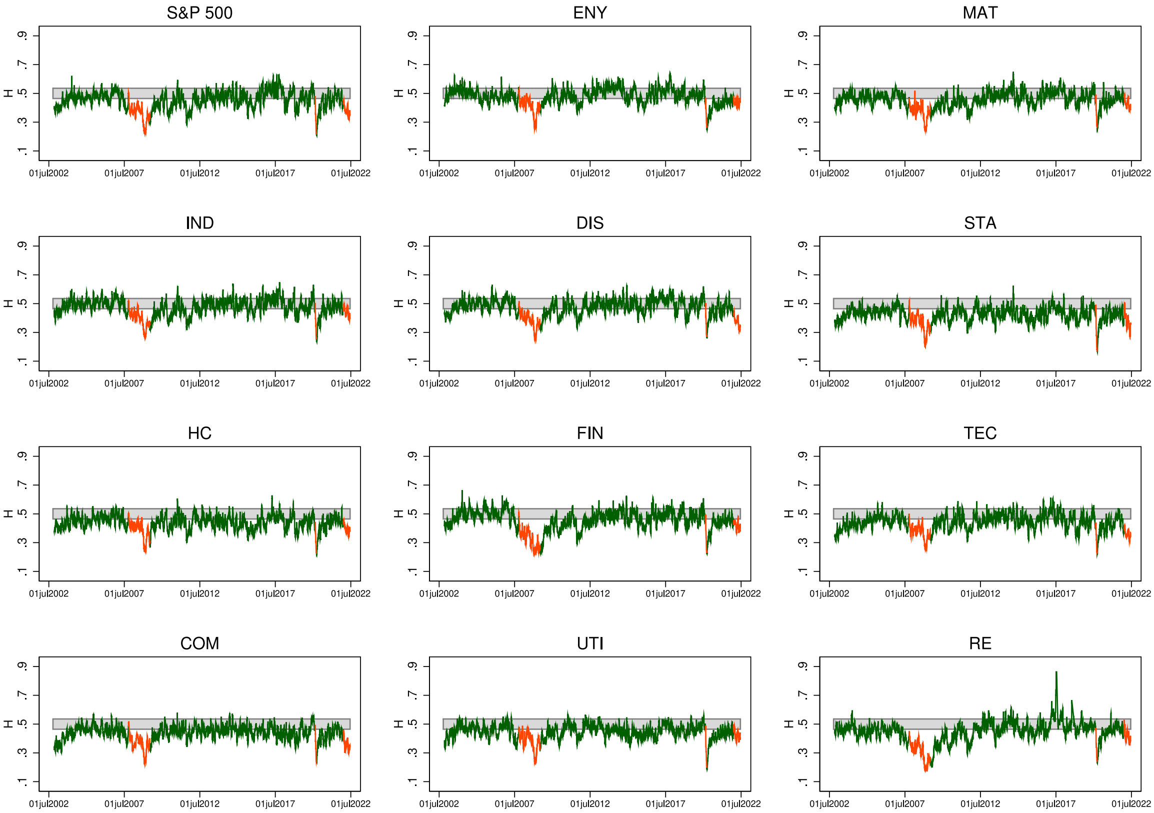


Fig. 2. Time-varying Hurst exponent for S&P 500 and sectorial indexes in the studied period. The light-gray area stands for $H = 0.5$ with a 95% confidence level. Green (red) lines indicates Bull (Bear) market regime. The plot shows significant deviations from the efficiency state. Under distress periods, these deviations are more pronounced.

Secondly, we can observe that the Hurst exponent differs between the different sectors and the S&P500. Below, we perform an econometric analysis to quantitatively measure this difference. Thirdly, it is evident that in periods of declines in the respective market indices (bear market), sectoral markets tend to be less efficient. These last two points can be seen more clearly, and in a more quantitative way in Fig. 3 where the left panel shows the Hurst exponent of the sectors for the bearish state, with values well below 0.5, outside the confidence interval, while in the right panel, the bullish state, market efficiency is clearly a more plausible hypothesis, with values close to 0.5 and within the confidence interval.

Fig. 4 shows the Pearson correlation coefficient between the Hurst exponents of the different sectors, distinguishing between bearish and bullish times. It is clear that, in general, the Hurst exponents of sector indices are correlated with each other, and with the Hurst exponent of the S&P 500. However, this correlation becomes much stronger in bear markets, precisely when markets are less efficient.

5.2. Statistical analysis

Several statistical analyses are developed for the Hurst time series of the S&P 500 and the sector indexes. A normality test, a Dickey–Fuller test to check if it is a stationary process, a t-test to test whether the market efficiency of the sector indexes is equivalent to the efficiency of the aggregate market, that of the S&P 500, and finally Granger causality tests. The results are presented in Table 3.

Table 3 panel (a) shows that the hypothesis that the Hurst exponents of each sectoral index are normally distributed is rejected. Panel (b) shows that the null hypothesis of a unit root is rejected for all Hurst exponents of the sectoral indices (i.e., all sectoral efficiency indices are stationary processes). Panel (c) shows that the means of the Hurst exponents of the S&P500 and every sector are statistically different from each other at any level greater than 1%. In other words, the measure of market efficiency is not a good proxy for sectoral efficiency. According to the Granger test showed in panel (d), the null hypothesis that the Hurst exponents of some sectors does not Granger-cause S&P500 cannot be rejected, in particular for sectors DIS, FIN, COM *y* RE at 1% significance, FIN and RE at 5% significance, and just FIN at 10% significance. These results make sense in terms of the financial and commodity crises faced by the market during the period under analysis. In the same way, the market efficiency (Hurst exponent of S&P500) affects, Granger-cause, the Hurst exponent of sectors STA, HC and TEC, since as shown in panel (e) the null hypothesis cannot be rejected for these sectors.

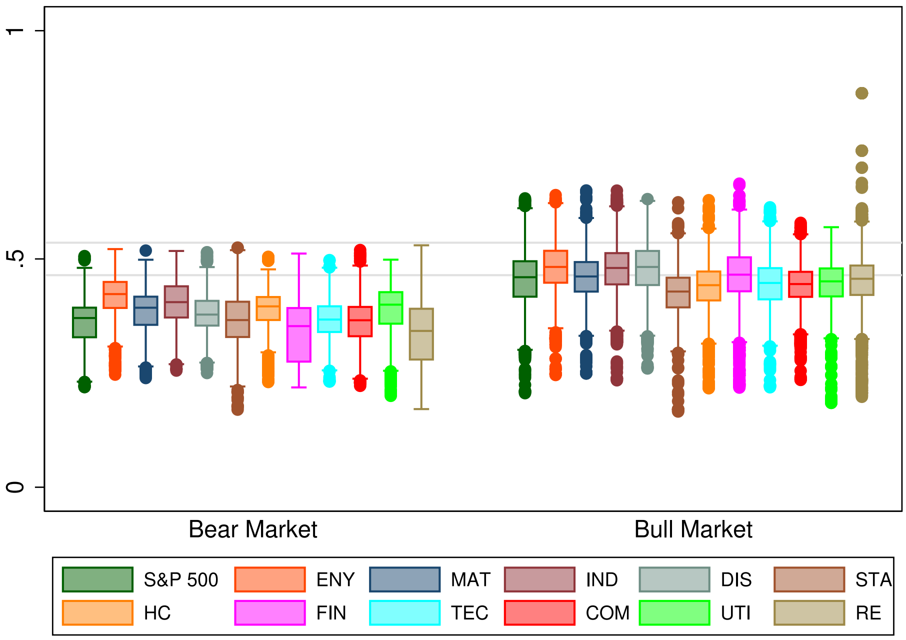
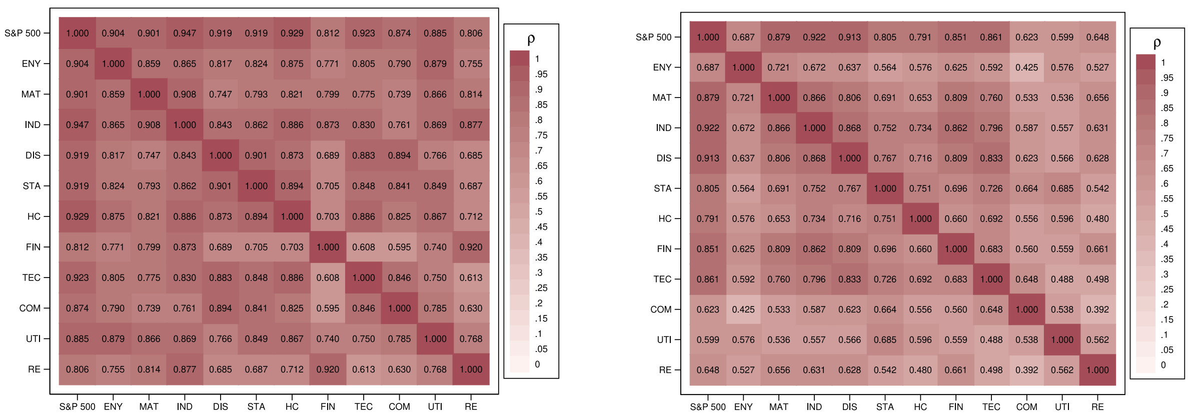


Fig. 3. Time-varying Hurst exponent Box-Plot for S&P 500 and sectorial indexes in the studied period. Light-gray parallel lines represents the limits of the 95% CI for $H = 0.5$. The comparison between bear and bull markets shows how the inefficiency state is enhanced during distress periods, where the time-varying Hurst exponent is mostly under 0.5. Otherwise, for boom periods, even though the Hurst exponents differ along sectors, it is distributed around the efficiency state.



(a) Bear Markets (b) Bull Markets

Fig. 4. Correlation among Hurst functions . The correlation rises in bearish times.

5.3. Econometric analysis

In this section, we measure the magnitude of the relationship between sectoral and aggregate market efficiency (S&P 500). A positive coefficient indicates that as aggregate market efficiency increases (decreases), sector market efficiency also increases (decreases). In this line, a coefficient close to one would indicate that sector market efficiency is almost the same as the aggregate market efficiency, in other words, both variables maintain an almost perfect correlation. On the other hand, a positive coefficient, but different from one, implies a positive correlation but a different level of market efficiency. A simple linear regression is used, which relates the sector market efficiency measure (the Hurst exponent of the particular sector) with the aggregate market efficiency measure (the Hurst exponent of the S&P 500):

$$H_{i,t} = \alpha_i + \beta_i H_{S\&P,t} + \epsilon_t \quad (2)$$

Table 3
Different test for the Hurst exponents of the S&P500 and every sector. Both normality and unit-root hypothesis for the Hurst exponents are rejected, while the t-test reveals how the mean of both S\&P and sectoral Hurst values are statistically different. Moreover, the Wald statistic cannot reject Granger-causality in the Hurst indexes from some sectors to the market (DIS, FIN, COM, RE) and from the market to particular sectors (STA, HC, TEC).

a) Shapiro--Wilk W test for normal data

	H_ENY	H_MAT	H_IND	H_DIS	H_STA	H_HC	H_FIN	H_TEC	H_COM	H_UTI	H_RE
Prob>z	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

b) Dickey-Fuller test for unit root - MacKinnon approximate p-palue

	H_ENY	H_MAT	H_IND	H_DIS	H_STA	H_HC	H_FIN	H_TEC	H_COM	H_UTI	H_RE
p-value for Z(t)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

c) ttest - Ha: mean(diff wrt H_SP500) != 0

	H_ENY	H_MAT	H_IND	H_DIS	H_STA	H_HC	H_FIN	H_TEC	H_COM	H_UTI	H_RE
Pr(|T| > |t|)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

d) Granger causality Wald tests - The Hurst exponents of each sector Granger-cause the Hurst exponent S&P500

	H_ENY	H_MAT	H_IND	H_DIS	H_STA	H_HC	H_FIN	H_TEC	H_COM	H_UTI	H_RE
Prob > chi2	0.001	0.0000	0.0000	**0.032**	0.0000	0.001	**0.329**	0.002	**0.019**	0.004	**0.084**

e) Granger causality Wald tests - The Hurst exponent of the S&P500 Granger-cause the Hurst exponents of each sector

	H_ENY	H_MAT	H_IND	H_DIS	H_STA	H_HC	H_FIN	H_TEC	H_COM	H_UTI	H_RE
Prob > chi2	0.002	0.0000	0.001	0.003	**0.128**	**0.242**	0.001	**0.510**	0.0000	0.009	0.0000

where $H_{i,t}$ and $H_t^{S\&P}$ represent the ith-sectorial and S&P 500 Hurst exponent, respectively, at time t. ε_t is the error of the regression. To establish a significant long-run relationship between these variables, the autoregressive distributed lag (ARDL) approach is used for cointegration analysis, given its advantages over alternative econometric strategies, see Kripfganz and Schneider (2018), Pesaran et al. (2001). The ARDL model in error correction (EC) representation can be described as:

$$\Delta H_{i,t} = b_0 - c_i \left(H_{i,t-1} + \beta_i H_{S\&P,t-1} \right) + \sum_{j=1}^{p-1} \psi_{i,j} \Delta H_{i,t-j} + \omega_i \Delta H_{S\&P,t} + \sum_{j=1}^{q-1} \theta_{i,j} \Delta H_{S\&P,t-j} + u_t \quad (3)$$

The variable $\theta_i = \sum_{j=2}^{q} \theta_{i,j}$ describes the short-run effect of the market Hurst function over the $i-$sectorial Hurst function, while the long-run relationship is calculated by $\beta_i = \theta_i / \left(1 - \sum_{j=2}^{p} \psi_{i,j}\right)$.

Table 4 indicates the short and long-run coefficients, θ_i and β_i respectively. From these econometric results we can extract the following conclusions. Firstly, it is clear from the adjustment coefficients, negative and significant, that all models cointegrate, showing a statistically significant long-term relationship between the sector market efficiency and the aggregate market efficiency. Secondly, long-run coefficients show a stronger relationship than the coefficients in the short-run, between the sector market efficiency and the aggregate market efficiency. Thirdly, there are sectors with almost a perfect correlation with the market efficiency index, i.e. a coefficient near to one, such as: IND (0.85), DS (0.98), FIN (0.98), and RE (1), while there are others with relatively lower relationships: ENY (0.6), COM (0.44), and UTI (0.55).

6. Concluding remarks

A multi-fractional Brownian approach was used to measure the level of different economic sectors market efficiency through the Hurst exponent. Using twenty years of S&P 500 data, our analysis showed that each sector has a particular level of market efficiency, and it cannot be statistically represented by the aggregate market efficiency. Besides, there is a long-term relationship between the sector market efficiency and the aggregate market efficiency. However, long-run coefficients show a stronger relationship than the short-run coefficients, between the sector market efficiency and the aggregate market efficiency. We observe sectors with almost a perfect correlation with the market efficiency index, i.e. a coefficient near to one, such as: IND (0.85), DS (0.98), FIN (0.98), and RE (1), while there are others with relatively lower relationships: ENY (0.6), COM (0.44), and UTI (0.55). If we restrict the analysis to periods of crisis, market efficiency by sector decreases sharply and the cross-correlation of efficiency between sectors tends to increase, Otherwise, during the non-crisis periods the market efficiency could be considered on average a good hypothesis, being the Hurst exponent close to 0.5, although it varies across sectors, being for some sectors even better than the aggregate market efficiency (S&P 500). In terms of cross-effects, the Hurst exponents of FIN and RE Granger-cause the Hurst exponent of S&P500 at 5% significance. In the same way, the market efficiency (Hurst exponent of S&P500) affects, Granger-cause, the Hurst exponent of sectors STA, HC and TEC.

Given the robustness of these results, it seems more correct to calculate sectoral market efficiency, rather than aggregate market efficiency.

Table 4
Error correction model (Eq. (3)) coefficients modeling cointegration among Hurst function for the market and every sectorial index . The results support cointegration in all the models, showing a stronger relation in the long-run than in the short-run coefficients run between sectoral efficiency and aggregate market efficiency.

	(1) D.H_ENY	(2) D.H_MAT	(3) D.H_IND	(4) D.H_DIS	(5) D.H_STA	(6) D.H_HC	(7) D.H_FIN	(8) D.H_TEC	(9) D.H_COM	(10) D.H_UTI	(11) D.H_RE
ADJ											
L.H_i	-0.0349***	-0.0770***	-0.0946***	-0.0452***	-0.0593***	-0.0539***	-0.0376***	-0.0512***	-0.0439***	-0.0354***	-0.0308***
	(0.00656)	(0.00884)	(0.00883)	(0.00897)	(0.00833)	(0.00745)	(0.00606)	(0.00962)	(0.00607)	(0.00734)	(0.00562)
LR											
L.H_SP500	0.602***	0.790***	0.854***	0.989***	0.728***	0.709***	0.986***	0.757***	0.449***	0.573***	1.034***
	(0.110)	(0.0438)	(0.0321)	(0.0728)	(0.0655)	(0.0641)	(0.0883)	(0.0762)	(0.0865)	(0.124)	(0.147)
SR											
D.H_SP500	0.451***	0.703***	0.808***	0.741***	0.632***	0.585***	0.742***	0.762***	0.385***	0.284***	0.419***
	(0.0262)	(0.0233)	(0.0209)	(0.0220)	(0.0268)	(0.0239)	(0.0230)	(0.0264)	(0.0256)	(0.0304)	(0.0305)
_cons	0.00725***	0.00796***	0.00822***	0.00154	0.00534**	0.00642***	0.000254	0.00462*	0.0107***	0.00651**	-0.000834
	(0.00203)	(0.00176)	(0.00154)	(0.00156)	(0.00196)	(0.00183)	(0.00149)	(0.00193)	(0.00195)	(0.00239)	(0.00201)
N	844	844	844	844	844	844	844	844	844	844	831
adj. R-sq	0.282	0.552	0.662	0.582	0.425	0.432	0.567	0.537	0.255	0.113	0.202

Standard errors in parentheses
"* p<0.05 ** p<0.01 *** p<0.001"

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CRediT authorship contribution statement

Marcelo J. Villena: Conceptualization, Data curation, Formal analysis, Investigation, Writing – original draft, Writing – review & editing, Methodology. **Axel A. Araneda:** Data curation, Formal analysis, Investigation, Methodology, Writing – original draft, Writing – review & editing, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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